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# REJECTION FACTOR OF DIFFERENCE AMPLIFIERS 

## BY

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## Summary

By analyzing the ordinary triode difference amplifier it is shown that the rejection factor can be made arbitrarily large, without necessity of pre-selection of valves or stringent mutual equality of corresponding circuit elements. Some circuits are given which guarantee a high rejection factor, even with $10 \%$ difference of the corresponding components of the two halves. The theory is verified by a number of measurements.

## Résumé

On montre, par l'analyse de l'amplificateur de tension différentielle ordinaire à triodes, que le facteur de rejection peut être arbitrairement élevé sans qu'il soit nécessaire de selectionner les tubes ou d'ajuster les éléments correspondants du circuit. Quelques circuits sont donnés qui assurent un facteur de réjection tout en permettant des tolérances de $10 \%$ pour les élêments correspondants des deux sections. La théorie fut vérifiée à l'aide de quelques mesures.

## Zusammenfassung

Durch Analyse des normalen Trioden-Differentialverstärkers wird gezeigt, daß der Schwächungsfaktor („rejection factor") beliebig groß gemacht werden kann, ohne daß eine Vorauswahl von Röhren oder eine gegenseitige Gleichheit entsprechende Schaltelemente erforderlich ist. Es werden einige Schaltungen angegeben, die einen hohen Schwächungsfaktor garantieren, wobei Abweichungen von 10\% der entsprechenden Schaltelemente der beiden Hälften zulässig sind. Die Theorie wird durch einige Messungen bestätigt.

## 1. Introduction

In some experiments it is necessary to measure small potential differences between two points while both points have in common a large potential difference with respect to earth, which is of no importance for the measurement. This purpose is served by the difference amplifier which has the property of amplifying anti-phase signals normally and in-phase signals hardly if at all. The question to which this article seeks to give an answer is in how far "hardly if at all" can approach "not at all". The basic circuit of a difference amplifier is given in fig. 1. If equal in-phase signals (i.e. $V_{g}=V_{g}$ )
are applied to the control grids the valves will work in parallel and resistor $\boldsymbol{R}_{\boldsymbol{c}}$ causes negative feedback. With equal anti-phase signals on the grids ( $V_{g}=-V_{g}$ ) the changes of current through $R_{c}$ almost cancel each other out so that the valves amplify without negative feedback. Apparently, the amplification for anti-phase signals can thus be made much larger than for in-phase signals.

If the two parts of the circuit were identical, equal signals on the control grids would give rise to equal signals on the anodes so that, assuming the same applied to all the following stages, no signal would be recorded at the output terminals. However, when the two parts of the circuit are not quite identical, equal signals on the control grids will cause signals on the anodes which are not exactly equal. Let these signals be $V_{a}$ and $V_{a}^{\prime}$ respectively. By putting $V_{1}=\frac{1}{2}\left(V_{a}+V_{a}^{\prime}\right)$ and $V_{2}=\frac{1}{2}\left(V_{a}-V_{a}^{\prime}\right)$, it can be said that under these circumstances the two anodes carry equal in-phase signals $V_{1}$ and equal anti-phase signals $V_{2}$.


Fig. 1. Ordinary difference amplifier with triodes.

The latter can in no way be distinguished from a desired anti-phase signal on the anodes caused by anti-phase signals on the control grids. From this follows that the quality of a difference amplifier can be indicated by the "rejection factor" $H$ which is defined as *)

$$
\begin{equation*}
H=\frac{\left(V_{a}-V_{a}^{\prime}\right)_{V g}=-V g^{\prime}=e}{\left(V_{a}-V_{a}\right)_{V g}=V_{g^{\prime}}=e} \tag{1}
\end{equation*}
$$

$H$ can also be regared as the ratio of in-phase signals and anti-phase signals at the input terminals which cause the same anti-phase signal at the output terminals.

In addition to $H$, the term "discrimination factor" $(F)$ is frequently encountered, and can be defined as follows:

[^0]\[

$$
\begin{equation*}
F=\frac{\left(V_{a}-V_{a}^{\prime}\right)_{V g=-V g^{\prime}=e}}{\left(V_{a}+V_{a}^{\prime}\right)_{V g=}=V_{g^{\prime}}=e} \tag{2}
\end{equation*}
$$

\]

that is,

$$
F=\frac{\text { total anti-phase amplification }}{\text { total in-phase amplification }} .
$$

In the next section the relations between the rejection factor and the discrimination factor of the total amplifier and those of the separate stages will be derived. From this analysis it follows that the problem of guaranteeing a large $H$-value for the total amplifier can be reduced to solving the problem of making $H$ for the first stage as large as possible. The latter problem constitutes the subject of the remainder of this article. In section 3 it is shown that, in order to make the rejection factor of a single stage large, two conditions should be fulfilled, namely a large $\mu$-value and a large impedance between the common point of the circuit and earth. In sections 4 and 5 some solutions for these requirements are given.

## 2. Rejection factor of a multi-stage amplifier

First a two-stage amplifier without feedback between the two stages will be considered: input signals of first stage $V_{g_{1}}, V_{g_{1}}^{\prime}$; output signals of first stage: $V_{a 1}, V_{a 1}^{\prime}$. Analogously for the second stage: input signals $V_{g 2}, V_{g 2}^{\prime}$ and output signals: $V_{a 2}, V_{a 2}^{\prime}$.

Let the following equations describe the two stages:
and

$$
\begin{aligned}
& V_{a 1}-V_{a 1}^{\prime}=a_{1}\left(V_{g_{1}}-V_{g_{1}}^{\prime}\right)+\beta_{1}\left(V_{g_{1}}+V_{g_{1}}^{\prime}\right) \\
& V_{a 1}+V_{a 1}^{\prime}=\gamma_{1}\left(V_{g_{1}}-V_{g_{1}}^{\prime}\right)+\delta_{1}\left(V_{g_{1}}+V_{g_{1}}^{\prime}\right)
\end{aligned}
$$

$$
\left.\begin{array}{l}
V_{a 2}-V_{a 2}^{\prime}=\alpha_{2}\left(V_{g 2}-V_{g 2}^{\prime}\right)+\beta_{2}\left(V_{g 2}+V_{g 2}^{\prime}\right), \\
V_{a 2}+V_{a 2}^{\prime}=\gamma_{2}\left(V_{g 2}-V_{g 2}^{\prime}\right)+\delta_{2}\left(V_{g 2}+V_{g 2}^{\prime}\right) . \tag{3}
\end{array}\right\}
$$

In these formulae $\alpha$ stands for the anti-phase amplification, $\alpha / \beta=H$ and $\alpha / \delta=F$ according to eqs (1) and (2), while

$$
G=\frac{\alpha}{\gamma}=\frac{\left(V_{a}-V_{a}^{\prime}\right)_{V g=-V g^{\prime}=e}}{\left(V_{a}+V_{a}^{\prime}\right)_{V g=-V g^{\prime}=e}}
$$

is a measure for the extent to which an anti-phase signal at the input causes an in-phase at the output.

Supposing $V_{g_{2}}=V_{a_{1}}$ and $V_{g_{2}}^{\prime}=V_{a_{2}}^{\prime}$ there follows:

$$
\left.\begin{array}{l}
V_{a_{2}}-V_{a_{2}}^{\prime}=\left(\alpha_{1} \alpha_{2}+\gamma_{1} \beta_{2}\right)\left(V_{g_{1}}-V_{g_{1}}^{\prime}\right)+\left(\beta_{1} \alpha_{2}+\delta_{1} \beta_{2}\right)\left(V_{g_{1}}+V_{b 1}^{\prime}\right), \\
V_{a 2}+V_{a_{2}}^{\prime}=\left(\alpha_{1} \gamma_{2}+\gamma_{1} \delta_{2}\right)\left(V_{g_{1}}-V_{g_{1}}^{\prime}\right)+\left(\beta_{1} \gamma_{2}+\delta_{1} \delta_{2}\right)\left(V_{g_{1}}+V_{g_{1}}^{\prime}\right),
\end{array}\right\}
$$

so that
$\alpha_{i}=$ anti-phase amplification of total amplifier $=\alpha_{1} \alpha_{2}\left(1+\frac{1}{G_{1} H_{2}}\right)$,
$H_{t}=$ rejection factor of total amplifier $=\frac{1+\frac{1}{G_{1} H_{2}}}{\frac{1}{H_{1}}+\frac{1}{F_{1} H_{2}}}$,

$$
\begin{equation*}
F_{t}=\frac{1+\frac{1}{G_{1} H_{2}}}{\frac{1}{H_{1} G_{2}}+\frac{1}{F_{1} F_{2}}} \tag{4c}
\end{equation*}
$$

and

$$
\begin{equation*}
G_{t}=\frac{1+\frac{1}{G_{1} H_{2}}}{\frac{1}{G_{2}}+\frac{1}{G_{1} H_{2}}} \tag{4~d}
\end{equation*}
$$

These relations can be simpified since for all circuits which will be considered the quantity $G$ is of the order of 10 . Even with a simple difference amplifier for the second stage, the magnitude of $H_{2}$ can easily be made large compared with unity (e.g. 100), so the term $1 / G_{1} H_{2}$ may be neglected with respect to 1 . Thus: $\alpha_{t}=\alpha_{1} \alpha_{2}, H_{t}=\frac{1}{\frac{1}{H_{1}}+\frac{1}{F_{1} H_{2}}}, F_{t}=\frac{1}{\frac{1}{H_{1} G_{2}}+\frac{1}{F_{1} F_{2}}}$ and $G_{t}=\frac{1}{\frac{1}{G_{2}}+\frac{1}{G_{1} F_{2}}}$.

In order to make $H_{t}$ large both $H_{1}$ and the product $F_{1} H_{2}$ should be made as large as possible. For the amplifiers dealt with in the next sections $F$ is about one order smaller than $H$, so that, by taking such an amplifier for the first stage and taking into account that $H_{2} \approx 100$, it follows that $H_{i}$ will be approximately equal to $H_{1}$.

For the simple amplifier, used for the second stage, $F$ is of the same order as $H$ while $G$ is, again, approximately 10 . Therefore $F_{l}$ will be larger than $H_{1}\left(=H_{t}\right)$ and $G_{t} \approx 10$.

From this it follows that for a multi-stage armplifier the total rejection factor will be nearly equal to that of the first stage if for the third and subsequent stages difference amplifiers are used for which the factors $H$ and $F$ have minimu $n$ values of about 20 and $G$ a minimum value larger than 2 , a condition which is easily fulfilled.

Thus the problem of designing a difference amplifier with a high rejection factor reduces to that of designing a first stage with a high rejection factor and a high discrimination factor.

It should be noted that the results obtained apply only to difference amplifiers with which no feedback exists between the stages.

## 3. Triode difference amplifier

First an analysis of the triode difference amplifier of fig. 2 will be given. The valves will be characterized by: $\mu=$ amplification factor, $S_{1}=$ transconductance and $S_{a}=R_{p}^{-1}$ with $R_{p}=$ plate resistance. For changes of currents and potentials the following equations apply:

$$
\begin{aligned}
i_{a} & =S_{1}\left(V_{g}-V_{c}\right)+S_{a}\left(V_{a}-V_{c}\right), V_{a}=-i_{a} R_{a} \\
i_{a}^{\prime} & =S_{1}^{\prime}\left(V^{\prime}-V_{c}\right)+S_{a}^{\prime}\left(V_{a}^{\prime}-V_{c}\right), V_{a}^{\prime \prime}=-i_{a}^{\prime} R_{a}^{\prime} \\
V_{c} & =\left(i_{a}+i_{a}^{\prime}\right) R_{c}
\end{aligned}
$$

From these equations $V_{a}-V_{a}^{\prime}$ and $V_{a}+V_{a}^{\prime}$ can be found as functions of $V_{g}$ and $V_{g}^{\prime}$ :

$$
\left.V_{a} \pm V_{a}^{\prime}=\frac{\left|\begin{array}{ccc}
V_{g} & \pm\left(\frac{S_{a}+R_{a}^{-1}}{S_{1}}\right) & \left(1+\mu^{-1}\right)  \tag{5}\\
V_{g}^{\prime} & -\left(\frac{S_{a}^{\prime}+R_{a}^{\prime-1}}{S_{1}^{\prime}}\right) & \left(1+\mu^{\prime-1}\right) \\
0 & \left(R_{a}^{\prime-1} \mp R_{a}^{-1}\right) & R_{c}^{-1}
\end{array}\right|}{\left\lvert\,-\left(\frac{S_{a}+R_{a}^{-1}}{S_{1}}\right)\right.} \begin{array}{ccc}
0 & \left(1+\mu^{-1}\right) \\
0 & -\left(\frac{S_{a}^{\prime}+R_{a}^{\prime-1}}{S_{1}^{\prime}}\right) & \left(1+\mu^{\prime-1}\right) \\
R_{a}^{\prime-1} & R_{c}^{-1}
\end{array} \right\rvert\, .
$$



Fig. 2. Ordinary difference amplifiers with triodes.

In deriving $H, F$ and $G$ from these formulae, the differençe between two corresponding quantities will be denoted by $\Delta$ (e.g., $\mu^{\prime}-\mu=\Delta \mu$ ), while the following approximations are made:

$$
\begin{aligned}
\mu+\mu^{\prime}=2 \mu, & \mu \mu^{\prime}=\mu^{2}, \\
S_{1}+S_{1}=2 S_{1}, & S_{1} S_{1}^{\prime}=S_{1}^{2}, \quad R_{a}+R_{a}^{\prime}=2 R_{a}, \\
R_{a} R_{a}^{\prime}=R_{a}^{2}, & \mu \gg 1, \quad S_{1} R_{a} \gg 1, \quad S_{1} R_{c} \gg 1 .
\end{aligned}
$$

In that case one finds, approx.

$$
\begin{align*}
H & =\frac{4}{\left(\frac{\Delta S_{1}}{S_{1}}+\frac{\Delta R_{a}}{R_{a}}\right) \frac{1}{S_{1} R_{c}}+\frac{1}{\mu}\left(\frac{\Delta \mu}{\mu}\right)\left(2+\frac{R_{a}}{R_{c}}\right)},  \tag{6a}\\
F & =\frac{2 S_{1} R_{c}}{1+S_{a} R_{a}}=\frac{2 \mu R_{c}}{R_{p}+R_{a}},  \tag{6b}\\
G & =\frac{4}{\left(\frac{\Delta S_{1}}{S_{1}}+\frac{\Delta R_{a}}{R_{a}}\right) \frac{1}{S_{1} R_{c}}+\frac{1}{\mu}\left(\frac{\Delta \mu}{\mu}\right) \frac{R_{a}}{R_{c}}+\frac{\Delta R_{a}}{R_{a}} \cdot 2} . \tag{6c}
\end{align*}
$$

In order to determine the lowest value that these factors might acquire, the following assumptions are made:
For all quantities involved $\Delta x / x \leqslant \delta\left(x=\mu, S\right.$, or $\left.R_{a}\right)$ applies, while the deviations are supposed to add in such a way that the denominators of (6a) and (6c) obtain the largest possible value:

$$
\begin{align*}
& H_{\min }=\frac{4}{\delta\left\{\frac{2}{S_{1} R_{c}}+\frac{1}{\mu}\left(2+\frac{R_{a}}{R_{c}}\right)\right\}}, \\
& F_{\min }=F=\frac{2 \mu}{\frac{R_{p}}{R_{c}}+\frac{R_{a}}{R_{c}}},  \tag{6b'}\\
& G_{\min }=\frac{4}{\delta\left(\frac{2}{S_{1} R_{c}}+\frac{R_{a}}{\mu R_{c}}+2\right)}
\end{align*}
$$

From these formulae it can be seen that a larger value of $R_{c}$ results in larger values of $H_{\text {min }}, F_{\min }$ and $G_{\min }$. The limiting values are:

$$
\left(H_{\min }\right)_{R_{c}=\infty}=\frac{2 \mu}{\delta}, \quad(F)_{R_{c}=\infty}=\infty, \quad \text { and } \quad\left(G_{\min }\right)_{R_{c}=\infty}=\frac{2}{\delta} .
$$

From (6a) it follows that fur a large $H_{\text {min }}$ one should use valves with a large amplification factor $\mu$. As $R_{c}$ will be finite, however, a large $\mu$-value alone does not guarantee a large rejection factor, but $S_{1} R_{c}$ should be of about the same order as $\mu$.

As $R_{c}$ can easily be made much larger than $R_{a}$ the formulae for $H_{\min }$ and $F$ can be reduced to

$$
\begin{equation*}
H_{\min }=\frac{2}{\delta\left(\frac{1}{S_{1} R_{c}}+\frac{1}{\mu}\right)} \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
F=2 S_{1} R_{c} \tag{7a}
\end{equation*}
$$

The principle underlying the circuit of fig. 2 offers two advantages, namely a large $F$ as well as a large $H$.

The circuits dealt with in the next sections will therefore be based on this same principle. The following can be said of all these circuits: In order to make $F$ large the impedance between the common point of the two halves of the circuit ( $C$ in fig. 2) and earth should be made as large as possible. To guarantee a large $H_{\text {min }}$ two conditions should be fulfilled:
(1) As for guaranteeing a large $F$ the impedance between the common point and earth should be made as large as possible.
(2) The ratio of the influence of the voltage of the common cathode point on the valve current and the influence of the input grid voltage on this current should be made as nearly equal to unity as possible.
If, in general:
$\boldsymbol{R}_{c}=$ total impedance between the common point and earth,
$S_{1}=\Delta$ (valve current): $\Delta$ (voltage between input grid and common point),
$1+1 / \mu=$ the above-mentioned ratio,
one obtains for all the circuits to be dealt with for $H_{\text {min }}$ the expression given in eq. (7)

In the next section some circuits with a large value of $\mu$ will be given, while section 5 deals with the problem of making $R_{c}$ large. It will appear that the second condition is harder to fulfill than the first, thus in formula (7):

$$
\frac{1}{S_{1} R_{c}} \gg \frac{1}{\mu} \quad \text { or } \quad H_{\min }=\frac{2 S_{1} R_{c}}{\delta}
$$

As $F$ is nearly equal to $2 S_{1} R_{c}$ (7a), $H_{\min }=F / \delta$. Taking $\delta=0 \cdot 1$, this leads to $H_{\text {min }}=10 F$, a result of which use was made in section 1 .

In the case of a large $R_{c}$, the quantity $G_{\min }$ will be equal to $2 / \delta=20$ (with $\delta=0 \cdot 1$ ), so that the condition $G_{\min }>10$, as used in section 1 , is easily satisfied.

In order to obtain numerical values $\delta$ will be taken equal to $0 \cdot 1$, thus

$$
\begin{equation*}
H_{\min }=\frac{20}{\frac{1}{S_{1} R_{c}}+\frac{1}{\mu}} \tag{7b}
\end{equation*}
$$

## 4. Circuits with large $\mu$

## Pentode circuits

With a pentode the influence of the anode voltage on the cathode current is much smaller than with a triode. However, the influence of the screengrid voltage is of the same order of magnitude as that of the anode voltage in a triode, but the advantage with a pentode is the possibility of decoupling the screen-grid to the cathode (fig. 3). Denoting the amplification factor from first grid to anode by $\mu_{1 a}$ and the transconductance from first grid to anode by $S_{1 a}, H_{\text {min }}$ is given by

$$
H_{\min } \approx \frac{20}{\frac{1}{R_{c} S_{1 a}}+\frac{1}{\mu_{1 a}}}
$$

In order to make $H_{\min }$ equal to $10 \mu_{1 a} \approx 2.10^{4}, R_{c}$ should be $1 / S_{1 a}=R_{p}$ with $R_{p}=$ plate resistance of the pentode. The magnitude of $R_{c}$ must, therefore, be one megohm or more; as a consequence no ordinary resistor can be used for $R_{c}$ in view of the d.c. voltage drop to be expected over such a resistor.


Fig. 3. Difference amplifier with pentodes.

An associated problem of a large impedance in the common cathode lead, is the maintenance of the required d.c. voltage between the screen-grids and the common cathode.

Before giving some solutions to these problems (section 5) it is desirable to deal first with a triode circuit which offers important advantages over the pentode circuit and also appreciably simplifies the decoupling of the auxiliary grids to the common point.

## Cascode circuit

First consider the "cascode" circuit (fig. 4) in which voltage changes with
respect to earth are applied to the various electrodes as indicated. The variation of the anode current $i$ can be calculated to be

$$
i\left(1+\frac{S_{a 1}}{S_{2}+S_{a 2}}\right)=S_{1} V_{g}-\left(S_{1}+S_{a 1}\right) V_{c}+\frac{S_{a 1} S_{2}}{S_{2}+\dot{S}_{a 2}} V_{g 2}+\frac{S_{a 1} S_{a 2}}{S_{2}+S_{a 2}} V_{a}
$$

with $S_{a 1}=S_{1} / \mu_{1}$ and $S_{a 2}=S_{2} / \mu_{2}$.


Fig. 4. Cascode of 2 triodes.

If the grid $g_{2}$ follows the cathode, $V_{g 2}$ is equal to $V_{c}$; thus the coefficient of $V_{c}$ in the formula becomes, in this case,

$$
\left(S_{1}+S_{a 1}-\frac{S_{a 1} S_{2}}{S_{2}+S_{a 2}}\right)
$$

Consequently, if we consider a difference amplifier analogous to that of fig. 2, but with the triodes replaced by two-triode cascodes and the grids of


Fig. 5. Difference amplifier with cascodes.
the upper triodes decoupled with respect to the common cathode (fig. 5), the $\mu$-value, as defined in the previous section, becomes:

$$
\begin{equation*}
\mu_{\mathrm{casc}}=\frac{\mu_{1}}{\left(1-\frac{\mu_{2}}{\mu_{2}+1}\right)} \approx \mu_{1} \mu_{2} \tag{8}
\end{equation*}
$$

Should the voltage of the upper grids follow the common cathode voltage only partly $\left(V_{g_{2}}=k V_{c}\right)$ the $\mu$-value becomes:

$$
\begin{equation*}
\mu_{\mathrm{casc}}=\frac{\mu_{1}}{\left(1-k \frac{\mu_{2}}{\mu_{2}+1}\right)} \tag{8a}
\end{equation*}
$$

With variations $\left|\frac{\Delta \mu_{1}}{\mu_{1}}\right| \leqslant 0 \cdot 1$ and $\left|\frac{\Delta \mu_{2}}{\mu_{2}}\right| \leqslant 0 \cdot 1$ (and $k=1$ ) the resulting variation for the cascode is found to be $\left|\frac{\Delta \mu_{\text {casc }}}{\mu_{\text {casc }}}\right| \leqslant 0.2$. It is possible, of course, to place more than two triodes in cascode which results in a still higher $\mu$-value.

In general one finds for a cascode consisting of $n$ triodes, whereby the voltage of the auxiliary grids follow the common point voltage completely,

$$
\mu_{\mathrm{casc}}=\mu_{1} \mu_{2} \ldots \mu_{n}, \text { and }\left|\frac{\Delta \mu_{\text {casc }}}{\mu_{\text {casc }}}\right| \leqslant(1 \cdot 1)^{n}-1
$$

The minimum rejection factor for a difference amplifier consisting of twotriode cascodes is thus given by

$$
H_{\min }=\frac{20}{\frac{1}{S_{1} R_{c}}+\frac{2}{\mu_{\text {caso }}}}
$$


and if $k=1$ (complete decoupling),

$$
H_{\min }=\frac{20}{\frac{1}{S_{1} R_{c}}+\frac{2}{\mu_{1} \mu_{2}}}
$$

With the cascode circuit, the difference in $\mu$-values clearly no longer limits the rejection factor. As the $\mu$-value can theoretically be made very large, the value of $S R_{c}$ will ultimately decide the value of $H_{\min }$. The contention ${ }^{1}$ ) that $R_{c}$ needs only to be made of the same order of magnitude as the anode-load resistors is therefore no longer valid if one wishes to achieve very high rejection factors.

## 5. Circuits for high cathode impedances

The total cathode impedance is made up by the following parts:
(a) The impedance between cathode and negative supply lead.
(b) The decoupling of the screen-grids in a pentode circuit or the control grids of the upper triodes in a cascode circuit generally impairs an impedance between the cathode and the positive supply lead.
(c) The resistance between cathode and filament; stray capacitances between the cathode and earth.

Although it is possible to enlarge these parts together by means of feedback, only methods to enlarge them separately will be given as most difference amplifiers are used for low-frequency applications. The impedances mentioned under (c) are the most difficult to control, but are, luckily, very high for low frequencies. For example the total capacitance between the cathode and earth, negative and positive supply leads and the filament together can be kept smaller than 10 pf . Of the resistance between cathode and filament it is often stated that this might obtain a low value. However for 60 valves of the types ECC 81, ECC 82 and E80CC the lowest value measured for this resistance was about $60 \mathrm{M} \Omega\left(V_{f}=6 V_{\text {eff }}, V_{c f} \approx 20 \mathrm{~V}\right)$. With increasing filament voltage the resistance decreases rapidly.

The methods to make the impedances $(a)$ and $(b)$ large are given below.

## Circuits for a high common-cathode impedance

The common-cathode resistor has to be made so large that the use of current-stabilizing circuits is indispensable. One of the circuits in common use is given in fig. 6 ; here the voltages $V_{g_{1}}$ and $V_{g 2}$ are kept constant. For the differential resistance $\Delta V_{a} / \Delta i_{a}$ is found approximatively

$$
\frac{\Delta V_{a}}{\Delta i_{a}}=R_{p}\left(1-i_{a} / i_{c}\right)
$$



Fig. 6. Pentode as current-source; $V_{g 1}$ and $V_{g 2}$ constant.


Fig. 7. Pentode as current-source; $V_{g_{1}}$ constant, $g_{2}$ decoupled to the cathode.
with $R_{p}=$ plate resistance and $i_{a}$ and $i_{c}$ equal to the anode and the cathode current of the pentode respectively, so that $\Delta V_{a} / \Delta i_{a}$ can amount to $10 \mathrm{M} \Omega$ maximum.

It is possible to effect an improvement of this circuit if the screen-grid voltage $V_{g_{2}}$ is not kept constant with respect to earth but constant with respect to the cathode. For low-frequency applications this can be done, for example, in the manner indicated in fig. 7. In this case:

$$
\frac{\Delta V_{a}}{\Delta i_{a}} \approx \mu_{1 a} R_{s}
$$

with $\mu_{1 a}=$ amplification factor of control grid to anode,

$$
R_{s}^{-1}=R_{k}^{-1}+R^{-1}
$$

In this way apparent resistances of several tens of megohms can be achieved.

Almost unlimitedly high resistances can be obtained by making use of a triode-cascode circuit. An example of this is given in fig. 8. Keeping the voltages of the grids constant with respect to earth, the differential resistance is given by:

$$
\begin{aligned}
& \frac{\Delta V_{a}}{\Delta i_{a}}>\mu_{1} \mu_{2} \mu_{3} r_{k}, \text { thus when using identical triodes: } \\
& \frac{\Delta V_{a}}{\Delta i_{a}}>\mu^{3} r_{k}
\end{aligned}
$$



Fig. 8. Cascode of triodes as current-source.

Analogously one finds for the case that $n$ identical triodes are placed above one another:

$$
\frac{\Delta V_{a}}{\Delta i_{a}}>\mu^{n} r_{k}
$$

The advantage of this method is the fact that with the cascode no screengrid current occurs so that the cathode current equals the anode current. The ultimate limitation of $\Delta V_{a} / \Delta i_{a}$ is determined by the control-grid current, and for very high rejection factors it is necessary to keep this current as small as possible.

## Decoupling of grids

As indicated in section 4, some grids have to be decoupled with respect to the common cathode. As a consequence, the impedance over which the respective grids receive the d.c. voltage is, for a.c., directly in parallel to the high cathode resistance. The d.c. voltage supply of those grids should therefore be designed carefully. We will next consider four different d.c. supply systems.

## (1) Floating d.c. voltage supply

One possibility for obtaining a constant voltage difference between the grids and the common cathode is the use of a separate supply, which should be "floating" with respect to earth. In this case no ohmic resistance is placed in parallel with $R_{c}$ but the latter is shunted by the capacitance between the supply and earth. This method will therefore be useful only for d.c. and l.f. applications. Another possibility for floating supply is a dry battery. Apart from the normal objections associated with its use, this method also gives a fairly large capacitance parallel to $\boldsymbol{R}_{\boldsymbol{c}}$.
(2) Voltage stabilizers, fed from the normal positive supply-lead

This circuit (fig.9) increases the capacitance between the common cathode


Fig. 9. Stabilizer tube, fed from positive lead of remainder of the amplifier.


Fig. 10. Circuit for increasing $R$ (fig. 9) by use of cathode follower.
and earth only slightly but the resistance $R$ is now in parallel with the resistance $\boldsymbol{R}_{\boldsymbol{c}}$. It is obvious that with stabilizer-tubes carrying one or more mA an electronic current-stabilizing circuit must be used for $R$. In the circuit of fig. 10 the differential resistance $\Delta V_{g} / \Delta i$ is given by

$$
\Delta V_{g} / \Delta i \approx R S R_{s}
$$

where $S=$ transconductance of the tube, and

$$
R_{s}^{-1}=R_{c}^{-1}+R_{i}^{-1}+R_{L}^{-1}
$$

so that $R$ is effectively increased by a factor $S R_{s}$.
In this case, too, a "cascode" effect can be achieved by connecting two stages in series. A practical disadvantage is the number of stabilizer tubes necessary for applying the proper voltage levels on the different grids. A simple circuit*) which can bring about very high resistances is given in fig. 11. By a suitable choice of the resistance $R_{p}$, the voltage variation $\Delta V_{a 2}$ can be made nearly equal to $\Delta V_{g}$, giving an increase of the effective resistance as seen from point $G$. For this purpose, $\boldsymbol{R}_{\boldsymbol{p}}$ should be made approximately equal to $R_{k} / \mu$, whereby $\mu=$ the amplification factor of the valve 1. With this circuit effective resistances of $100 \mathrm{M} \Omega$ and more are obtained with $R=0.1 \mathrm{M} \Omega$.


Fig. 11. Circuit for increasing $R$ (fig. 9) by making use of amplifier, with an amplification of unity.

## (3) Cathode-follower circuit

If it is necessary, in connection with the high-frequency performance of the amplifier, to limit the capacitance of the common point to earth, the circuit of fig. 12 can be used. The common cathode of the amplifier should be connected with point $C$ and the screen or other auxiliary grids to $G$. The

[^1]extent to which the voltage of $G$ follows that of $C$ depends upon the cathodefollower operation of the circuit. It should be borne in mind that $R_{s}$ is effectively parallel to $R_{k}$, so that a possible increase of $R_{k}$ serves a useful purpose only when $R_{s}$ is correspondingly large. In any case $V_{G}$ is not entirely equal to $V_{C}$ so that $k$ (eq. (8a)) is not equal to unity. Nevertheless, this does not need to be a limitation for the minimum rejection factor. The circuit of fig. 11 without the resistance $R$ can also be used in which $C=$ grid of valve I and $G=$ anode of valve II. In this case $G$ follows $C$ almost completely.


Fig. 12. Circuit for decoupling the auxiliary grids by means of a cathode follower.

## (4) Decoupling by a capacitor

All methods described so far are suitable for d.c. amplifiers. If, however, this is not a requirement, considerable simplifications can often be made, such as decoupling of the grids to the common cathode by a capacitor, while a high-resistance voltage divider provides the correct d,c. voltage for the grids.

## 5. Measurements

In this section the results of some measurements will be given. The circuits shown are the less complicated ones.

Pentode circuit (fig. 13)
With switch $S$ in position 1, the screen-grids are fed from the positive supply lead through the resistance M22. This method is often applied in difference amplifiers. In position 2, a floating voltage source (a battery) keeps the screen-grid cathode voltage constant. In both positions the rejection factors of 15 different pairs of EF 40 valves were measured. The results are given in graph 1 . In this graph, $H_{0}$ is the rejection factor in position 1 and $H_{n}$ the rejection factor in position 2. Each point gives the $H_{0}$ - and $H_{n}$-values for a certain pair of valves. It thus appears thati $\left(H_{0}\right)_{\text {min }}$ $=360$ and $\left(H_{n}\right)_{\min }=18000$, an improvement of more than 50 times. The
measurements were made at a frequency of $250 \mathrm{c} / \mathrm{s}$. With increasing frequency, the rejection factor decreases rapidly, due to the influence of the capacity between battery and earth. At $2000 \mathrm{c} / \mathrm{s}$, the $H_{n}$-values were 4 times smaller.


Fig. 13. Difference amplifier with pentodes. The screen-grids are decoupled to the common point by a battery (position 2 of switch $S$ ) or fed from the positive lead through the resistor M22 (position 1 of switch $S$ ).


Graph 1. Rejection factors as measured for the circuit of fig. 13.
$H_{0}=$ rejection factor with switch $S$ in position $1, H_{\boldsymbol{n}}=$ rejection factor with switch $S$ in position 2.

Cascode circuit (fig. 14)
For the cascode two halves of the valve ECC81 $(\mu=60)$ were used. The resistance between the cascode and the negative supply lead was made large by using the circuit of fig. 8 , with 2 triodes in cascode. The impedance between the grids of the upper triodes and the positive supply lead was enlarged by using the circuit of fig. 10, which can be recognized in fig. 14.


Fig. 14. Difference amplifier with cascodes.

For the valve ECC81-1 various arbitrarily chosen valves were taken. For each the rejection factor was determined in this circuit $\left(H_{n}\right)$ and this was compared with the rejection factor for the same valve in the circuit which is obtained by short-circuiting the upper triodes $\left(H_{0}\right) \quad H_{0}$ is thus a measure of the rejection factor which could be obtained by the conventional circuits. The results are given in graph 2 The frequency at which the measurements took place was $250 \mathrm{c} / \mathrm{s}$. This measurement was repeated for different valves ECC81 in position ECC81-2. The lowest $H_{n}$ measured amounted to
40000. The valve for which this was the case had an $H_{0}$ of only 500 . With a margin of $10 \%$ in all parameters 1200 was calculated for $\left(H_{0}\right)_{\min }$ and 36000 for $\left(H_{n}\right)_{\min }$. The frequency dependence of $H_{n}$ was determined for various pairs of valves. The results for the two extreme cases are given in graph 3. Curve $a$ indicates the trend of $H_{n}$ with a moderate $H_{n}$ at low frequencies while graph $b$ refers to a valve with a very high $H_{n}$ at low frequencies. The large difference in trend might be explained by the fact that possible differences in valve and wiring capacitances make themselves be felt more severely in $\boldsymbol{b}$ than in $\boldsymbol{a}$.


Graph 2. Rejection factors as measured for the circuit of fig. 14.
$H_{0}=$ rejection factor with the valve ECC $81-2$ short-circuited, giving values one can expect for an ordinary triode amplifier.
$H_{n}=$ rejection factor for the circuit as drawn in fig. 14.



Graph 4. $H$ versus amplitude of in-phase signal.

Finally, a measurement was made of $H_{n}$ as a function of the magnitude of the in-phase signal. The result is reproduced in graph 4.

## Conclusion

It is possible to design difference amplifiers which guarantee high rejection factors without having to satisfy strenuous requirements concerning the mutual parity of valve and circuit parameters. The circuit of fig. 14 used as a low-frequency difference amplifier gives a rejection factor larger than 30000. Higher values of $H_{\min }$ can be realized by using cascode circuits consisting of more than two triodes. The ultimate limitation is determined by the magnitude of the total impedance between the common cathode and earth.

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[^0]:    *) This quantity is sometimes referred to as "rejection ratio", "anti-phase - in-phase ratio", "transmission factor" ${ }^{1}$ ) or "common-mode-rejection".

[^1]:    *) This circuit was suggested by Mr M. van Tol of this laboratory.

